Algebra II Final Exam

Multiple Choice
Identify the choice that best completes the statement or answers the question.

1. Suppose you drop a tennis ball from a height of 9 feet. After the ball hits the floor, it rebounds to 90% of its previous height. How high will the ball rebound after its third bounce? Round to the nearest tenth.
   a. 5.9 feet  
   b. 7.3 feet  
   c. 6.6 feet  
   d. 1.1 feet

2. Which of the following measures of center and variation describe the data set below:
   32, 38, 44, 62, 17, 29, 36, 45
   a. mean is 37.875 and population standard deviation is 13.22  
   b. mean is 37.875 and population standard deviation is 12.36  
   c. mean is 43.286 and population standard deviation is 13.22  
   d. mean is 43.286 and population standard deviation is 12.36

3. Let \( h(x) = 5x^3 + 2x^2 - 7x + 9 \) and \( f(x) = -4x^3 + 3x^2 + 2x + 1 \), which of the following equals \( h(x) + f(x) \):
   a. \( x^3 - x^2 - 5x - 8 \)  
   b. \( 9x^3 - x^2 - 9x + 8 \)  
   c. \( x^3 + 5x^2 - 5x + 10 \)  
   d. \( 9x^3 + 5x^2 - 9x + 1 \)
4. Ms. Blackshoe made a side-by-side distribution of test scores from her two classes. The distributions are separated by hour. Here are the distributions:

What can be said about these distributions?

a. First hour had a higher median score and IQR
b. First hour had a lower median score and IQR
c. First hour had a higher median score and a lower IQR
d. First hour had a lower median score and a higher IQR

5. Which family of functions does \( y = \sqrt{x - 3} + 2 \) belong to? What is the domain and range of the function?

a. It is a square root function  
   Domain: \( \{x | x > -3\} \), Range: \( \{y | y > 0\} \)
b. It is a square root function  
   Domain: \( \{x | x \geq 3\} \), Range: \( \{y | y \geq 2\} \)
c. It is a square function  
   Domain: \( \{x | x < -3\} \), Range: \( \{y | y \in \mathbb{R}\} \)
d. It is a square function  
   Domain: \( \{x | x \geq 3\} \), Range: \( \{y | y \geq 2\} \)

6. Solve \( \log(4x + 10) = 3 \).

a. \( \frac{7}{4} \)  
   c. 250
b. 247.5  
   d. 990
7. The table shows the results of a survey of students in two math classes.
Find \( P(\text{more than 1 hour of TV} \mid \text{6th period class}) \). Round to the nearest thousandth.

<table>
<thead>
<tr>
<th>Did You Watch More Than One Hour of TV Last Night?</th>
<th>Yes</th>
<th>No</th>
</tr>
</thead>
<tbody>
<tr>
<td>3rd period class</td>
<td>11</td>
<td>6</td>
</tr>
<tr>
<td>6th period class</td>
<td>13</td>
<td>10</td>
</tr>
</tbody>
</table>

a. 0.647  
b. 0.565  
c. 0.435  
d. 0.765

8. If a student tosses a fair coin four times. Each time she got a tails. What is the probability her next toss will also be a tail?

a. \( \frac{1}{4} \)  
b. \( \frac{1}{3} \)  
c. \( \frac{1}{2} \)  
d. 1

9. Evaluate \( \log 0.01 \)

a. \(-10\)  
b. \(-2\)  
c. 2  
d. 10

10. Using the graph below, write a polynomial function in factored form.

a. \( y = (x + 1)(x + 2)(x - 5) \)  
b. \( y = 0.25(x + 1)(x + 2)(x - 5) \)  
c. \( y = -0.25(x + 1)(x + 2)(x - 5) \)  
d. \( y = -(x + 1)(x + 2)(x - 5) \)
11. What is the inverse of the function \( y = \log_2(x + 5) \)?
   
   a. \( y = 2^x \)                  c. \( y = 2^x + 5 \)
   
   b. \( y = \log_{-2}(x - 5) \)       d. \( y = 2^x - 5 \)

12. Simplify the following: \((6 - 3i) - (2i - 8)\).
   
   a. \(-i - 2\)                c. \(-5i - 2\)
   
   b. \(-i + 14\)              d. \(-5i + 14\)

13. Solve the equation for \( x \):
   \[
   \frac{-2}{x + 4} = \frac{4}{x + 3}.
   \]
   
   a. \( \frac{13}{6} \)             c. \( \frac{8}{3} \)
   
   b. \(-11\)                      d. \( \frac{11}{3} \)

14. Multiply. State the excluded values:
   \[
   \frac{z^2 + 2z}{z + 1} \cdot \frac{z^2 + 3z + 2}{z^2 + 3z}.
   \]
   
   a. \( \frac{z^2 + 2z}{z + 3}, \ z \neq -1, -3 \)               c. \( \frac{z + 2}{z + 3}, \ z \neq -1, -3 \)
   
   b. \( \frac{z + 2}{z + 3}, \ z \neq -1, 0, -3 \)              d. \( \frac{z^2 + 2z}{z + 3}, \ z \neq -1, 0, -3 \)

15. Use the Change of Base Formula to solve \( 2^x = 90 \). Round to the nearest ten-thousandth.
   
   a. 7.6133                  c. 3.2459
   
   b. 9.3658                  d. 12.9837

16. Write the explicit formula for the sequence. Then find the fifth term in the sequence.
   \( a_1 = -5, r = 5 \).
   
   a. \( a_n = -5 \cdot (-5)^n; -3125 \)                        c. \( a_n = -5 \cdot (5)^n; -15625 \)
   
   b. \( a_n = 5 \cdot (-5)^{n-1}; 3125 \)                     d. \( a_n = -5 \cdot (5)^{n-1}; -3125 \)
17. The pH of a liquid is a measure of how acidic or basic it is. The concentration of hydrogen ions in a liquid is labeled \([H^+]\). Use the formula \(\text{pH} = -\log [H^+]\) to answer questions about pH.

Find the pH level, to the nearest tenth, of a liquid with \([H^+]\) about \(6.5 \times 10^{-3}\).

a. 3.8  
   b. 2.2  
   c. -3.8  
   d. 3.0

18. If all possible results are equally likely and mutually exclusive, what is the probability that a spin of the spinner will land on an upper case letter that is not a vowel?

a. 0.9  
   b. 0.7  
   c. 0.5  
   d. 0.3

19. Use Pascal’s Triangle to find the binomial coefficients: \(\binom{12}{4}\) and \(\binom{12}{9}\).

a. 495; 220  
   b. 99; 44  
   c. 11880; 79833600  
   d. 12; 495

20. Consider a normal distribution, what is true about the relationship between the mean and the median?

a. The distribution is symmetrical, and the mean is greater than the median.  
   b. The distribution is symmetrical, and the mean is smaller than the median.  
   c. The distribution is symmetrical, and the mean is equal to the median.  
   d. There is no way to tell from the information given.

21. The half-life of Carbon-14 is about 5730 years. It was determined that a bone specimen contained about 35% of Carbon-14. Which type of function models the number of years ago that this animal was alive?

a. Linear  
   b. Quadratic  
   c. Exponential  
   d. Logarithmic
22. Determine whether this graph represents a function. If it is a function, state the family it would belong to and find the domain and range.

![Graph Image]

a. Yes, it is a function, it is a quadratic
   Domain: \( \{ x \mid x \geq 0 \} \), Range: \( \{ y \mid y \in \mathbb{R} \} \)

b. Yes, it is a function, it is a square root
   Domain: \( \{ x \mid x \geq 0 \} \), Range: \( \{ y \mid y \in \mathbb{R} \} \)

c. No, it is not a function
   Domain: \( \{ x \mid x \geq 0 \} \), Range: \( \{ y \mid y \geq 0 \} \)

d. No, it is not a function.
   Domain: \( \{ x \mid x \geq 0 \} \), Range: \( \{ y \mid y \in \mathbb{R} \} \)

23. You are buying a car for $7,500. The bank offers a 5.5% annual percentage rate, compounded monthly for 5 years. You agree to make $350 payments. What sequence represents the value of this loan at the end of each of the first 4 months?

a. 7534.38, 7568.91, 7915.82, 7952.10
d. 7150, 6800, 6450, 6100

b. 7562.50, 7628.44, 7698.00, 7771.39
c. 7184.38, 6867.30, 6548.78, 6228.79

24. A couple has three children, what is the probability that they are all boys?

a. \( \frac{1}{3} \)

b. \( \frac{3}{8} \)

c. \( \frac{1}{8} \)

d. \( \frac{1}{6} \)
25. Using the normal distribution below, what percent is represented by the shaded region?

$\mu = 6, \sigma = 1.5$

a. 68%  

b. 99.7%  

c. 95%  

d. 97.5%

26. Determine the zeros and end behavior of the polynomial graph:

a. The zeros are -3, 2, 4 and the end behavior is left to right.

b. The zeros are 3, -2, -4 and the end behavior is left to right.

c. The zeros are -3, 2, 4 and the end behavior is $y$ approaches negative infinity as $x$ approaches negative infinity and $y$ approaches positive infinity as $x$ approaches positive infinity.

d. The zeros are -3, 2, 4 and the end behavior is $y$ approaches positive infinity as $x$ approaches negative infinity and $y$ approaches negative infinity as $x$ approaches positive infinity.
27. Determine what family of functions models the graph below, then write a function for the graph.

![Graph Image]

a. exponential, \( y = 0.5(2)^x \)  
   b. exponential, \( y = 2(0.5)^x \)  
   c. exponential, \( y = (2 \cdot 0.5)^x \)  
   d. logarithmic, \( y = 2(5)^x \)

28. Which table below does **not** represent a function?

<table>
<thead>
<tr>
<th>x</th>
<th>1</th>
<th>2</th>
<th>4</th>
<th>8</th>
</tr>
</thead>
<tbody>
<tr>
<td>y</td>
<td>1</td>
<td>5</td>
<td>4</td>
<td>7</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>x</th>
<th>1</th>
<th>2</th>
<th>1</th>
<th>2</th>
</tr>
</thead>
<tbody>
<tr>
<td>y</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>1</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>x</th>
<th>1</th>
<th>2</th>
<th>1</th>
<th>2</th>
</tr>
</thead>
<tbody>
<tr>
<td>y</td>
<td>4</td>
<td>5</td>
<td>6</td>
<td>7</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>x</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
</tr>
</thead>
<tbody>
<tr>
<td>y</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>1</td>
</tr>
</tbody>
</table>

29. Find the zeros of the function, \( y = x^3 - 2x^2 - 5x + 10 \).

a. \( \sqrt{5}, -\sqrt{5}, 2 \)  
   b. \( \sqrt{5}, -\sqrt{5}, -2 \)  
   c. \( 5, -5, 2 \)  
   d. \( 5, -5, -2 \)
30. Graph the equation $4x^2 + 9y^2 = 36$. Then describe the graph and its lines of symmetry.

a. The graph is an ellipse. The center is at the origin. It has two lines of symmetry, the $x$-axis and the $y$-axis.

b. The graph is an ellipse. The center is at the origin. It has two lines of symmetry, the $x$-axis and the $y$-axis.

c. The graph is an ellipse. The center is at the origin. Every line through the origin is a line of symmetry.

d. The graph is an ellipse. The center is at the origin. Every line through the origin is a line of symmetry.
31. The graph below is a transformation of the parent function \( f(x) = \frac{1}{x} \). Write the equation of the graph.

\[
a. \quad f(x) = \frac{1}{x-1} - 5 \\
b. \quad f(x) = \frac{1}{x+1} - 5 \\
c. \quad f(x) = \frac{-1}{x-1} - 5 \\
d. \quad f(x) = \frac{-1}{x+1} - 5
\]

32. Identify the conic section.

a. Parabola  
   b. Hyperbola  
   c. Circle  
   d. Ellipse

33. When using a control chart, which of the following is used to make the centerline:

a. median  
   b. mean  
   c. mode  
   d. standard deviation
34. If $R$ is the total resistance for a parallel circuit with two resistors of resistances $r_1$ and $r_2$, then
\[ \frac{1}{R} = \frac{1}{r_1} + \frac{1}{r_2}. \]
Find the resistance $r_1$ if the total resistance $R$ is 20 ohms and $r_2$ is 75 ohms. Round your answer to the nearest ohm if necessary.

- a. 16 ohms
- b. 1405 ohms
- c. 27 ohms
- d. 102 ohms

35. The amount of money in an account with continuously compounded interest is given by the formula
\[ A = Pe^{rt}, \]
where $P$ is the principal, $r$ is the annual interest rate, and $t$ is the time in years. Calculate to the nearest hundredth of a year how long it takes for an amount of money to double if interest is compounded continuously at 6.2%. Round to the nearest tenth.

- a. 1.1 yr
- b. 6.9 yr
- c. 11.2 yr
- d. 0.6 yr

36. Identify the center of the hyperbola with the equation
\[ \frac{(x + 3)^2}{25} - \frac{(y + 1)^2}{81} = 1. \]
Graph the hyperbola.

- a. center: (-3, -1)
- b. center: (-3, -1)
- c. center: (3, 1)
- d. center: (3, 1)
37. Write an equation of a circle with center (-3, -4) and radius 2.5.

a. \((x + 3)^2 + (y + 4)^2 = 6.25\)

b. \((x + 3)^2 + (y + 4)^2 = 2.5\)

c. \((x - 3)^2 + (y - 4)^2 = 6.25\)

d. \((x - 3)^2 + (y - 4)^2 = 2.5\)

38. If no digit appears more than once, how many 2-digit numbers can be formed from the digits 2, 3, 4, 5, 6, 7, 9? Does this situation represent a combination or a permutation?

a. 4, combination

b. 42, permutation

c. 21, combination

d. 97, permutation

39. Solve the following polynomial: \(4x^2 + 11x = -12\).

a. \(\frac{-11 \pm i\sqrt{313}}{8}\)

b. \(\frac{-11 \pm i\sqrt{71}}{8}\)

c. \(\frac{-11 \pm \sqrt{313}}{8}\)

d. \(\frac{-11 \pm \sqrt{71}}{8}\)

40. Divide \(4x^3 - 10x^2 + 8x\) by 2x.

a. \(2x^2 + 5x + 4\)

b. \(8x^4 - 20x^3 + 16x^2\)

c. \(2x^2 - 8x + 6\)

d. \(2x^2 - 5x + 4\)

41. Describe the vertical asymptote(s) and hole(s) for the graph of \(y = \frac{(x - 5)(x - 2)}{(x - 2)(x + 4)}\).

a. asymptote: \(x = -4\) and hole: \(x = 2\)

b. asymptotes: \(x = -4\) and \(x = 2\)

c. asymptote: \(x = -5\) and hole: \(x = -4\)

d. asymptote: \(x = 4\) and hole: \(x = -2\)

42. Use summation notation to write the series 50 + 52 + 54 + ... for 11 terms.

a. \(\sum_{n=1}^{11} (48 + 2n)\)

b. \(\sum_{n=1}^{48} (50 + 2n)\)

c. \(\sum_{n=1}^{10} (48 + 2n)\)

d. \(\sum_{n=1}^{11} (50 + 2n)\)
43. Use the tree diagram below to create the sample space from flipping 3 coins. Then determine the probability of flipping exactly 1 head.

- a. HHH, HHT, HTH, HTT, THH, THT, TTH, TTT; P(1H2T) = \( \frac{1}{8} \)
- b. HHH, HHT, HTH, HTT, THH, THT, TTH, TTT; P(1H2T) = \( \frac{8}{3} \)
- c. HHH, HHT, HTH, HTT, THH, THT, TTH, TTT; P(1H2T) = \( \frac{1}{3} \)
- d. HHH, HHT, HTH, HTT, THH, THT, TTH, TTT; P(1H2T) = \( \frac{3}{8} \)

44. \( x^2 - 8x - y + 19 = 0 \)
- a. parabola; vertex (4, -3)
- b. parabola; vertex (4, 3)
- c. parabola; vertex (-3, 4)
- d. parabola; vertex (3, 4)

45. Write the logarithmic expression as a single logarithm: \( 5 \log_b q + 2 \log_b y \)
- a. \( \log_b (q^5 y^2) \)
- b. \( (5 + 2) \log_b (q + y) \)
- c. \( \log_b (q^5 + y^2) \)
- d. \( \log_b \left(q^5 + \sqrt{y^2}ight) \)
46. Sketch the asymptotes and graph the function.

\[ y = \frac{x^2 - 7x + 12}{x^2 - 1} \]

a. 

![Graph of \( y = \frac{x^2 - 7x + 12}{x^2 - 1} \)]

b. 

![Graph of \( y = \frac{x^2 - 7x + 12}{x^2 - 1} \)]

c. 

![Graph of \( y = \frac{x^2 - 7x + 12}{x^2 - 1} \)]

d. 

![Graph of \( y = \frac{x^2 - 7x + 12}{x^2 - 1} \)]

47. Compare the graphs of the pair of functions. Describe the family of functions they belong to and then describe how the graph of the second function relates to the graph of the first function.

\( y = \log x \text{ and } y = \log(x - 1) + 2 \)

a. They are logarithmic functions
   The second function is the graph of \( y = \log x \) moved 1 left and 2 up.

b. They are trigonometric functions
   The second function is the graph of \( y = \log x \) moved 1 down and 2 right.

c. They are logarithmic functions
   The second function is the graph of \( y = \log x \) moved 1 right and 2 up.

d. They are exponential functions
   The second function is the graph of \( y = \log x \) moved 1 up and 2 right.
48. Let \( f(x) = 3x - 5 \) and \( g(x) = x^2 \), find \( g(f(-2)) \):

a. -121  

b. 121  

c. 7  

d. -17

49. Identify the logarithmic form of \( 10^{0.602} = 4 \)

a. \( \log_{10} 0.602 = \frac{1}{4} \)  

b. \( \log_{10} \frac{1}{4} = 0.602 \)  

c. \( \log_{10} 4 = -0.602 \)  

d. \( \log_{10} 4 = 0.602 \)

50. Which of the following is a continuous quantitative variable?

a. shoe size  

b. weight  

c. IQ  

d. number of pets owned

51. The table shows the predicted growth of a particular bacteria after various numbers of hours. Write an explicit formula for the sequence of the number of bacteria.

<table>
<thead>
<tr>
<th>Hours ((n))</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
</tr>
</thead>
<tbody>
<tr>
<td>Number of Bacteria</td>
<td>25</td>
<td>50</td>
<td>75</td>
<td>100</td>
<td>125</td>
</tr>
</tbody>
</table>

a. \( a_n = 25n \)  

b. \( a_n = n + 25 \)  

c. \( a_n = 25n + 25 \)  

d. \( a_n = \frac{1}{25} n \)

52. Write a recursive formula for the sequence 8, 11, 14, 17, 20, .... Then find the next term.

a. \( a_n = a_{n-1} - 3 \), where \( a_1 = 8 \); 23  

b. \( a_n = a_{n-1} + 3 \), where \( a_1 = 8 \); 23  

c. \( a_n = a_{n-1} + 3 \), where \( a_1 = 23 \); 8  

d. \( a_n = a_{n-1} - 3 \), where \( a_1 = 3 \); -7
53. Theodore is having lunch at a submarine sandwich shop. He may choose one of the following breads: whole wheat or rye. He may choose one of the following condiments: salad dressing, mayonnaise, or mustard. He may choose one of the following fillings: turkey, spicy tofu or chicken.

   a. Use the counting principle to determine the total number of possible sandwiches.

   b. Draw a tree diagram to show the many different sandwich choices he has, assuming he chooses one type of bread, one condiment, and one filling.

   c. Create a complete sample space of the different sandwiches possible.

   d. If he chooses at random from the complete menu, what is the probability that he will order turkey on rye with any condiment?

54. Using the equation: $x^2 + (y + 2)^2 = 49$; find the radius and center of the circle, then graph.
55. Nick bought a new car for $17,500. The value of the car is depreciating at a rate of 16% a year.
   
a. What family of functions would best represent this depreciation?
   
b. Define the variables and write an equation that models the situation.
   
c. Find the value of the car after 3.5 years.
   
d. How long before the car is worth half of its beginning value?

56. Use the drawing of a cube below to determine the following:
   
a. The surface area of the cube.
   
b. The volume of the cube.

![Cube Diagram]
57. The following is the data collected from a newspaper activity in Mr. Ranger’s seminar.


a. Construct a histogram of this data (use bin width of two).

b. Comment about the shape of the data collected.

c. Find the five number summary and IQR for the data set.

d. Construct a Boxplot of this data set.

58. Yvette is creating a design in which she draws squares within squares as shown. She begins with a square having a perimeter of 15 inches. Each inner square has a perimeter that is 90% of the perimeter of the previous square. Calculate the sum of the perimeters of a total of 8 rectangles. (Round to the nearest hundredth, if necessary.) Explain your method.
59. Match each function $f(x)$ with a function $g(x)$, so that $f(x)$ and $g(x)$ are inverses. Then determine $f(x) + g(x)$.

<table>
<thead>
<tr>
<th>$f(x)$</th>
<th>$g(x)$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$e^x$</td>
<td>$\frac{3}{x} + 2$</td>
</tr>
<tr>
<td>$(x + 2)^2$</td>
<td>$\sqrt{x - 5} + 2$</td>
</tr>
<tr>
<td>$(x - 2)^3$</td>
<td>$\ln x$</td>
</tr>
<tr>
<td>$\frac{1}{(x - 1)}$</td>
<td>$\frac{1}{x - 1}$</td>
</tr>
<tr>
<td>$(x - 2)^2 + 5$</td>
<td>$\sqrt{x - 2}$</td>
</tr>
<tr>
<td>$\frac{x + 1}{x}$</td>
<td>$\frac{x + 1}{x}$</td>
</tr>
</tbody>
</table>
60. Analyze the rational function \( f(x) \) and then sketch the graph.

\[
f(x) = \frac{x + 3}{x^2 - x - 12}
\]

a. Factor the numerator and denominator and find the domain of the function.

b. Find the x and y intercepts and holes.

c. Find the vertical and horizontal asymptotes.

d. Determine the end behavior of the function.

e. Sketch the function.